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HEAT TRANSFER FROM A SHORT HEATED SECTION IN ANNULAR FLOW

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Abstract—An experimental study was performed to investigate the heat transfer from a short, heated section of the core of a vertical, annular flow system. Among the parameters allowed to vary were the location and the heat flux of the heated section, types of fluid, Prandtl number of the fluid, and Reynolds number of the flow.

Test results showed higher heat-transfer coefficients when the heated section is located within a short distance from the flow entry. This distance is approximately one-sixth of the hydrodynamic entrance length for laminar flow and ten hydraulic diameters for turbulent flow. Beyond this distance, heat transfer closely resembles that of the flow normal to a cylinder, and test results have been correlated into a single curve.

NOMENCLATURE

C_p, specific heat [Btu/lb°F];
D_1, inner diameter (or core diameter) of the annulus [ft];
D_2, outer diameter of the annulus [ft];
D_H, hydraulic diameter of the annulus [ft];
h, convective heat-transfer coefficient [Btu/sft^2°F];
k, thermal conductivity [Btu/sft°F];
L, distance from the flow entry to the front end of the heated section or location of the heated section [ft];
L', length of the heated section [ft];
Nu, Nusselt number;
Pr, Prandtl number with properties evaluated at film temperature;
Pr_∞, Prandtl number with properties evaluated at bulk temperature;
Re, Reynolds number;
V, average velocity of the flow [ft/s].

Greek symbols
ρ, density [lb/ft^3];
μ, viscosity [lb/fts];
μ_w, viscosity, evaluated at the wall temperature of the heated section [lb/fts];
α, β γ, φ, Ψ, ζ, values as defined in content.

INTRODUCTION

ANNULAR flow refers to fluid flow along the axial direction between two concentric cylinders. As in all flow systems, the development of the velocity profile (or the boundary layer of the flow) requires a certain flow distance known as the hydrodynamic entrance region. When there is heat transfer through the duct wall of the flow system, the development of the temperature profile (or the thermal boundary layer) also requires a certain distance known as the thermal entrance region. The growth of these boundary layers is of primary concern to the heat-transfer characteristics of the flow system, since the mode of heat transfer depends very much on the thickness and flow pattern within these boundary layers as well as the heat or temperature distribution along the heat transfer surface.

Most studies of annular flow heat transfer have dealt with long heat-transfer sections. They were generally concerned with cases where either the heat transfer began at the flow entry, so that the hydrodynamic and thermal boundary layers commenced to develop simultaneously, or the heat-transfer section was preceded by a sufficiently long flow length that the velocity profile was fully developed before it passed over the heat-transfer surface.

The purpose of this work was to investigate the heat-transfer characteristics of an annular flow system where a short section of the annular core was heated. The geometry of the annulus was fixed. However, the location of the heated section was varied with respect to the flow entry. Other variables included the type of fluid, the properties (Prandtl number) of the fluid, the velocity (Reynolds number) of the flow, and the heat flux of the heated section.

Without considering the development of the boundary layers, this work would be similar to a study of the local heat-transfer coefficient of an annular flow system. The special feature of this work is that the development of the velocity profile could be disturbed, as the flow passed over the heated duct wall. The thermal boundary layer started to develop on the heated duct wall, but did not become completely developed since the length of the heated section was too short. Such a heat-transfer problem is unique, but its solution may have applications elsewhere. Furthermore, the scarcity of information on such a heat-transfer system presents a challenging research problem.

DESCRIPTION OF EXPERIMENTAL EQUIPMENT

Experiments were performed on a vertical, annular, upward flow system. A short section of the annular core was heated and its location could be varied with respect to the flow entry.

The major experimental equipment contained the following items:

(a) *Hydraulic loop.* A schematic diagram of the flow system is shown in Fig. 1. It consisted of a main tank, a calming tank, a test section, a pump, a flow meter, pipes and valves. The main tank had a capacity of 45 gallons and was provided with heat exchange coils (not shown on Fig. 1) to maintain constant fluid temperature. Pipes and fittings of 2 in dia. connected the main tank with the calming tank. Straighteners were provided inside the 2 in pipe leading to the calming tank to reduce the intensity of vortices that might have existed in the flow. The calming

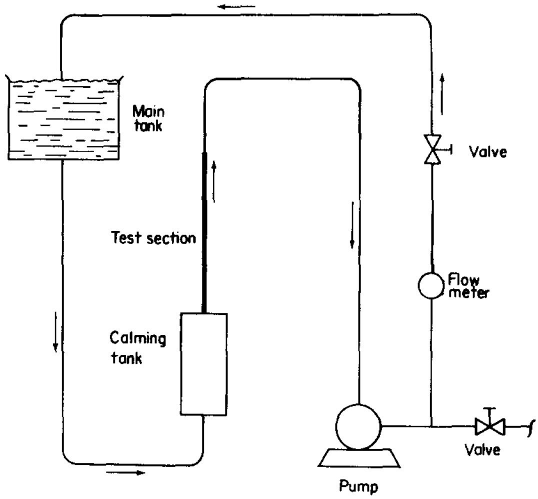

FIG. 1. Schematics of test system.

tank was 8 in. dia. and 18 in. high. After leaving the calming tank, the fluid flowed through the test section and then was pumped back to the main tank via a flow meter and valve.

(b) *Test section.* The test section, shown in Fig. 2, was 28 in long, 2 in pipe with 0·625 in core. The inner wall of the pipe was machined to a smooth finish with a diameter of 2·16 in. The entrance end of the pipe extended into the calming tank about 2 in. The heated section of the core was a 0·625 in. o.d. copper cylinder fitted snugly over a commercial heater. The length of the heated section was 1·60 in., which was approximately 2·5 times the core diameter or approximately one hydraulic diameter of the annulus. Teflon collars of 0·625 and 2 in. dia. long were provided on both ends of the heated section to reduce the heat loss due to conduction. The rest of the core of the annulus was made of 0·625 in. stainless steel tubes.

The location of the heated section is defined as the distance from the entrance to the 2 in. pipe to the front end of the heated section. Six different lengths of the upstream portion of the core (0·44, 2, 4, 7, 14·75 and 20·75 in.) were used to effect variation of the location of the heated section with respect to the flow entry. The location of the heated section could also be varied

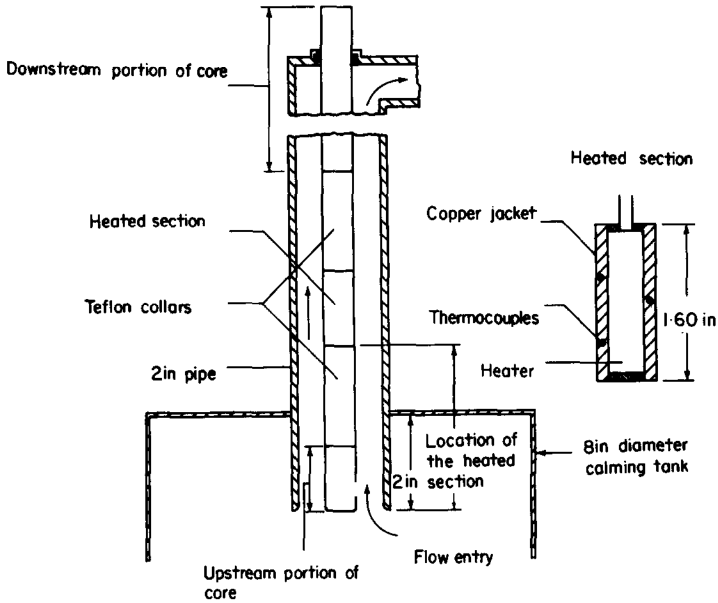

FIG. 2. Test section.

by moving the core assembly in an axial direction, when a sufficiently long upstream portion of the core was used. The downstream portion of the core was 27·50 in. long. The power supply to the heater and the thermocouple wires for measuring the wall temperatures of the heated section were passed inside of this tube. The core assembly was supported on the downstream end of the test section. However, three thin fins were used to center the core at the entrance of the test section. Effect of these fins on the heat transfer was found to be negligibly small.

(c) *Instrumentation.* Number 30 gauge copper–constantan thermocouples were used to measure temperatures. There were six thermocouple junctions. Three of the junctions were located inside the copper cylinder of the heated section to indicate the wall temperatures of the heated section. They were approximately 0·625 in. apart along the flow direction, and were located in three equally spaced directions (120° apart). The other three indicated the fluid temperatures upstream and downstream of the test section, and the ice point of water. Any one of the latter three junctions could be used as a reference temperature junction. Output of the thermocouples were either read on a millivoltmeter or recorded on a strip-chart recorder.

The heat flux of the heated section was varied by changing the power supplied to the heater. A variable transformer, a voltmeter and a wattmeter were used to facilitate the adjustments. Maximum capacity of the heater was 240 W at 120 V. The geometry of the heated section was such that it gave an average heat flux of approximately 156 Btu/h ft^2 for each watt of electrical power supplied to the heater.

EXPERIMENTAL PROCEDURE

Four fluids of different viscosity ranges were used as the test fluids—water, crude oil and two lubricating oils. The range of Prandtl number of these fluids is shown in Table 1.

For each test (after steady state had been reached) the flow rate, the location of the heated section, the power supplied to the heated section,

Table 1. Equipment specifications and ranges of test variables

Test Section	$D_1 = 0{\cdot}625$ in
	$D_2 = 2{\cdot}160$ in
	Length = 28 in
Heated Section	$D_1 = 0{\cdot}625$ in
	$L' = 1{\cdot}60$ in
	Heat flux 6260–32000 Btu/hft^2
	Location of the heated section 0–12 hydraulic diameters from flow entry.
Fluids	Water $2{\cdot}90 \leqslant Pr \leqslant 5{\cdot}51$
	Crude oil $12{\cdot}20 \leqslant Pr \leqslant 23{\cdot}70$
	Lubricating oil $44{\cdot}50 \leqslant Pr \leqslant 177$
	Lubricating oil $138{\cdot}0 \leqslant Pr \leqslant 329$
Flow Rate	Average velocity 0·19 to 1·90 ft/s.

and the thermocouple readings were recorded. This information was then combined with the physical properties of the fluid and the geometry of the test section to give the results in the following dimensionless forms:

Nusselt number, $Nu = \dfrac{hD_1}{k}$;

Prandtl number, $Pr = \dfrac{C_p\mu}{k}$;

Reynolds number, $Re = \dfrac{D_H V\rho}{\mu}$;

viscosity ratio, $\dfrac{\mu}{\mu_w}$;

dimensionless location of the heated section,

$\dfrac{L}{D_H}$;

dimensionless length of the heated section, $\dfrac{L'}{D_1}$;

Diameter ratio of the annulus, $\dfrac{D_2}{D_1}$.

In most cases, the change in the fluid temperature across the test section was small; therefore the fluid temperature upstream of the test section was used as the bulk temperature of the fluid. Furthermore, because the length of the heated section was short, the differences among the three wall temperatures of the heated section were small (less than 7 per cent for the extreme case tested) so that the arithmetic mean of these three was taken as the wall temperature.

All fluid properties, except μ_w were evaluated at the mean film temperature (one half of the sum of the wall temperature of the heated section and the bulk temperature of the fluid). The value of μ_w was evaluated at the wall temperature of the heated section.

Values of the convective heat-transfer coefficient h were determined from the average heat flux and the temperature difference between the wall of the heated section and the bulk fluid.

It should be pointed out that the core diameter of the annulus (D_1) was used as the characteristic length for both the Nusselt number and the dimensionless length of the heated section; while the hydraulic diameter of the annulus ($D_H = D_2 - D_1$) was used as the characteristic length for both the Reynolds number and the dimensionless location of the heated section. Both core diameter and the hydraulic diameter had been used as the characteristic length in calculating the Nusselt and Reynolds numbers. Mueller [9] and Davis [2] used the core diameter in both Nusselt number and Reynolds number. However, they indicated that their Reynolds number did not have the usual hydraulic meaning. On the other hand, using the hydraulic diameter for the Nusselt number could result in erroneous values when the outer diameter of the annulus is large compared with its core. Furthermore, the difference in the velocity gradients at the inner and outer wall of the annulus also discourages the use of the hydraulic diameter as a characteristic length in the Nusselt number.

In view of the preceding discussion and the fact that the Reynolds number and the location of the heated section are related to the hydraulics of the flow, the hydraulic diameter has been used as a characteristic length for both the Reynolds number and the dimensionless location of the heated section. Since the Nusselt number and the length of the heated section are related to the heat transfer of the heated section, the diameter of the heated section has been used as a characteristic length for both the Nusselt number and the dimensionless length of the heated section.

Specifications of the experimental equipment and the ranges of the test variables are summarized in Table 1.

PRESENTATION OF RESULTS

(a) *Effect of the location of the heated section on the heat transfer*

Boundary-layer theory predicts a higher heat-transfer coefficient near the flow entry where the boundary layer is relatively thin and just commencing to develop. The Nusselt number is therefore excepted to be larger when the heated section is located near the entry, and to decrease as the heated section moves downstream. The experimental results presented in Fig. 3 verify this prediction. However, the effect of location of the heated section on the Nusselt number becomes negligible (or the heat-transfer coefficient approaches its fully developed value) when the heated section moved beyond a certain critical distance from

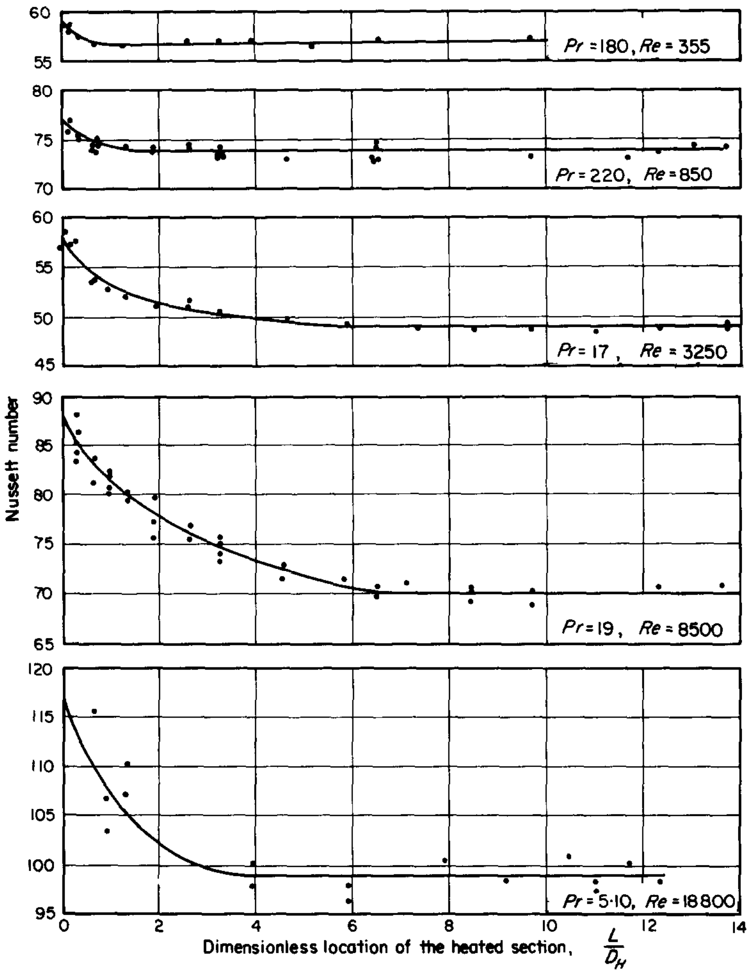

FIG. 3. Effect of the location of the heated section on heat-transfer heat flux = 9220 Btu/h ft^2.

the flow entry. This critical distance was shown to depend on the Reynolds number of the flow and, to a lesser extent, on the Prandtl number of the fluid. For laminar flow, the critical distance is approximately one-sixth of the hydrodynamic entrance length as calculated according to the analytical equation reported by Heaton, Reynolds and Kays [5]. And, for turbulent flow, the critical distance is less than ten hydraulic diameters for almost all turbulent flow cases investigated.

It is known that, for high-Prandtl-number fluids such as those used in this experiment, the thermal entrance length is much longer than the hydrodynamic entrance length. The special feature brought out by this experiment is that the unheated entrance length required for a short heated section to attain the fully-developed heat-transfer coefficient, in both laminar and turbulent flow, is very much shorter than the hydrodynamic entrance length. This characteristic feature makes the heat transfer of the short heated section a unique problem. Nevertheless, it should be pointed out that the critical distance could depend on the length of the heated section and the diameter ratio of the annulus; however, these were not included in the present work.

(b) *Effect of flow rate, fluid properties, and heat flux on heat transfer*

The effects of variations in the flow rate, fluid properties and heat flux on the heat transfer are shown on Figs 4–7, where the Nusselt number is plotted against the Reynolds number. All of the data shown on these graphs were taken in a region where the Nusselt number was not influenced by the location of the heated section. The Nusselt number increases as the Reynolds number is increased. However, the rate of increase differs from different ranges of the Reynolds number. For low Reynolds numbers, the Nusselt number seems to be proportional to $Re^{0.25}$, while for high Reynolds numbers, the Nusselt number seems to be proportional to $Re^{0.80}$. The increasing rate of change in the Nusselt number with the increase in Reynolds number is definitely associated with the change in the flow pattern and the velocity gradient within the boundary layer. The transition of flow pattern, however, does not produce noticeable fluctuation of the heat-transfer coefficient as is usually experienced by the pressure gradient. The flow regime can therefore only be identified through the Reynolds number.

An increase in the heat flux essentially

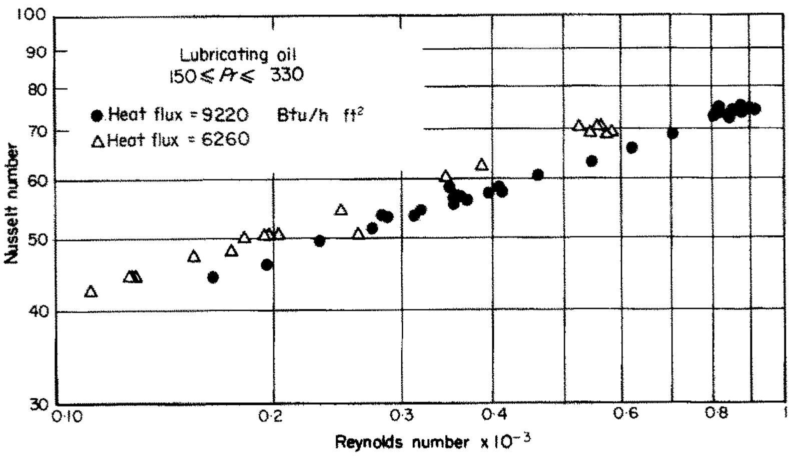

FIG. 4. Heat transfer of short heated section in annulus (lubricating oil $Pr_{\infty} \sim 350$).

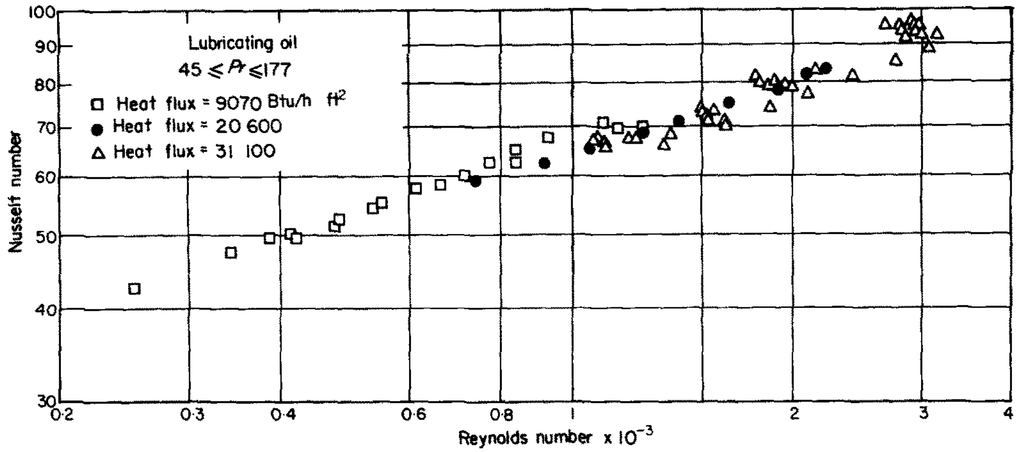

FIG. 5. Heat transfer of short heated section in annulus (lubricating oil $Pr_\infty \sim 182$).

increases the wall temperature of the heated section, and hence the film temperature of the fluid. Among the fluid properties, viscosity is the one most sensitive to temperature change. It decreases with increases in the film temperature. Apparently, a change in viscosity affects the Reynolds number more than it does the heat-transfer coefficient. Therefore as heat flux increases, the curves of Nusselt number vs. Reynolds number are shifted slightly in the direction of higher Reynolds number. However, for a given Reynolds number, the change in Nusselt number is relatively small as compared with the change in the heat flux.

(c) *Correlation of the experimental results*

The relationship between the Nusselt number and the Reynolds number shown in Figs. 4–7 takes the general form of heat transfer in pipe flow. However, probably because of the short length of the heated section, the relationship closely resembles that of flow normal to a cylinder [3]. The experimental data shown on these graphs have been correlated to a single curve (Fig. 8) which can be expressed as

$$Nu = \zeta Re^{\beta} Pr^{0.3} \left(\frac{\mu_w}{\mu}\right)^{0.14}$$

where the values of ζ and β are functions of

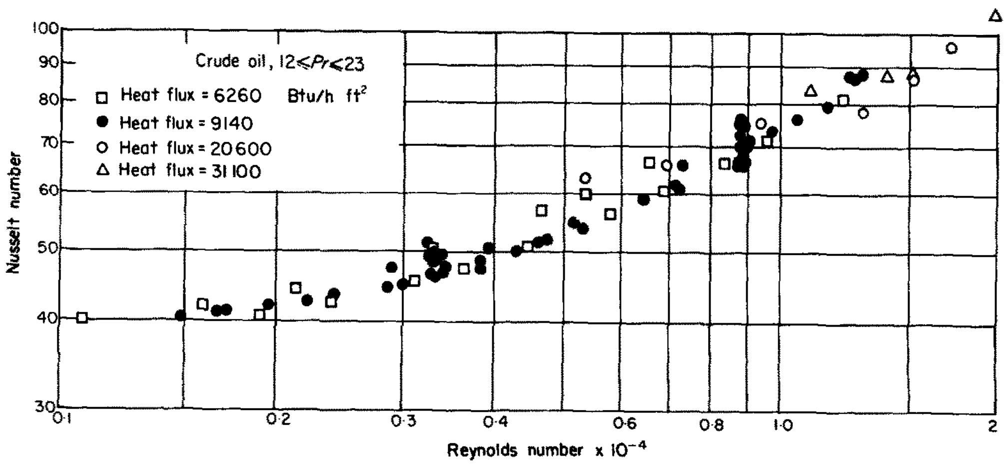

FIG. 6. Heat transfer of short heated section in annulus (crude oil, $Pr_\infty \sim 25$).

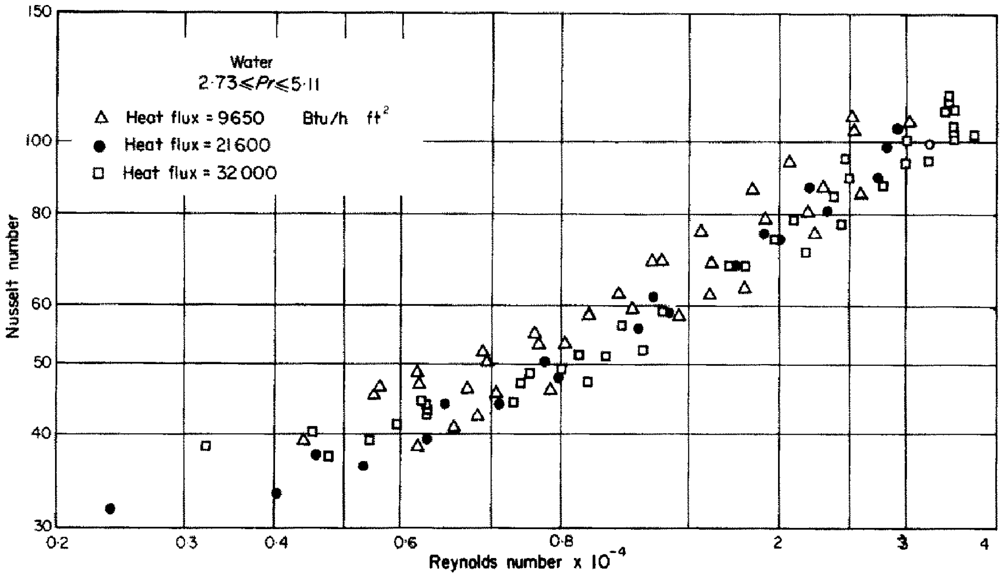

FIG. 7. Heat transfer of short heated section in annulus (water, $Pr_{\infty} \sim 5{\cdot}50$).

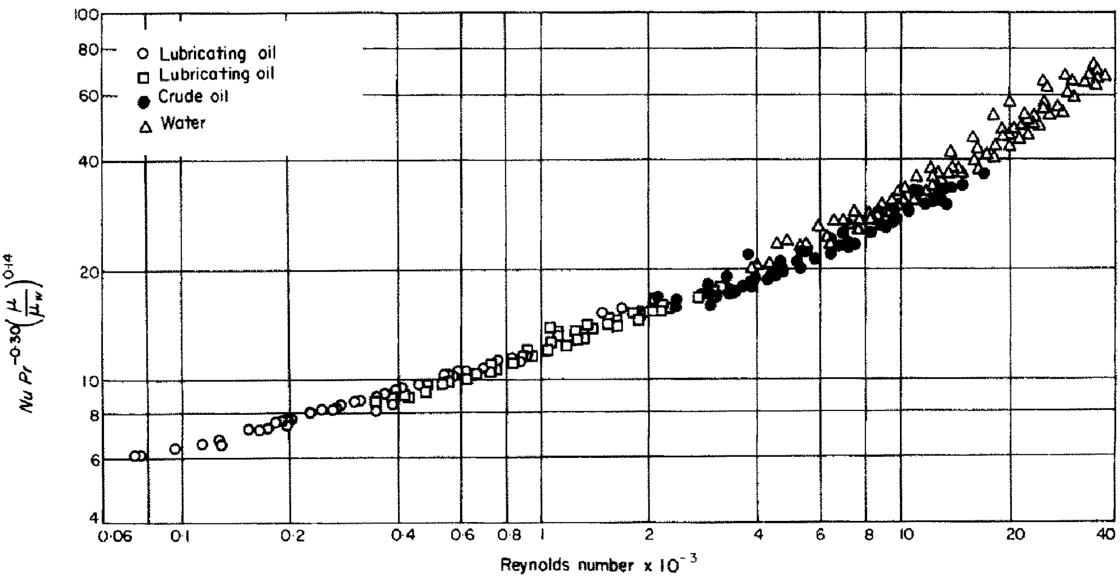

FIG. 8. Heat transfer of short heated section in annulus (all fluids).

the Reynolds number shown in the following table:

Reynolds number	ζ	β
70–800	2·05	0·25
700–6000	1·15	0·35
5000–15000	0·30	0·48
15000–40000	0·035	0·70

Spread of the experimental data was within ± 8 per cent of this correlation.

The effects of the length of the heated section, the diameter ratio of the annulus, and possibly the buoyancy force, on the heat-transfer coefficient could be included in "ζ". Although no effort in this work has been made to find the functional dependence of ζ on these parameters, the experimental results are compared with those results where the length of the heated section and the diameter ratio of the annulus vary.

COMPARISON OF RESULTS

Since only limited investigations have been conducted with short heated sections, some comparisons are màde with the results of long heated sections.

Test results showed that as long as the heated section is located beyond the critical distance heat transfer is independent of location, and the critical distance is much shorter than the hydrodynamic entrance length. It is, therefore, interesting to compare the heat-transfer result of this work with that obtained downstream of a hydrodynamically developed flow.

Heat transfer of hydrodynamically developed annular flow with long heated section has been studied by Lundberg, McCuen, and Reynolds [7] in the laminar flow regime, and by Kays and Leung [6] in the turbulent flow regime. Analytical results of Lundberg *et al.*, indicated that the Nusselt number in laminar flow is a function of the diameter ratio of the annulus and a dimensionless parameter, L/D_H $1/Pr\,Re$. Kays and Leung showed the Nusselt number of the turbulent flow is a function of the Prandtl number and Reynolds number. Comparison between the present results and these analytical solutions are shown on Figs. 9 and 10. (Note that the Nusselt number on

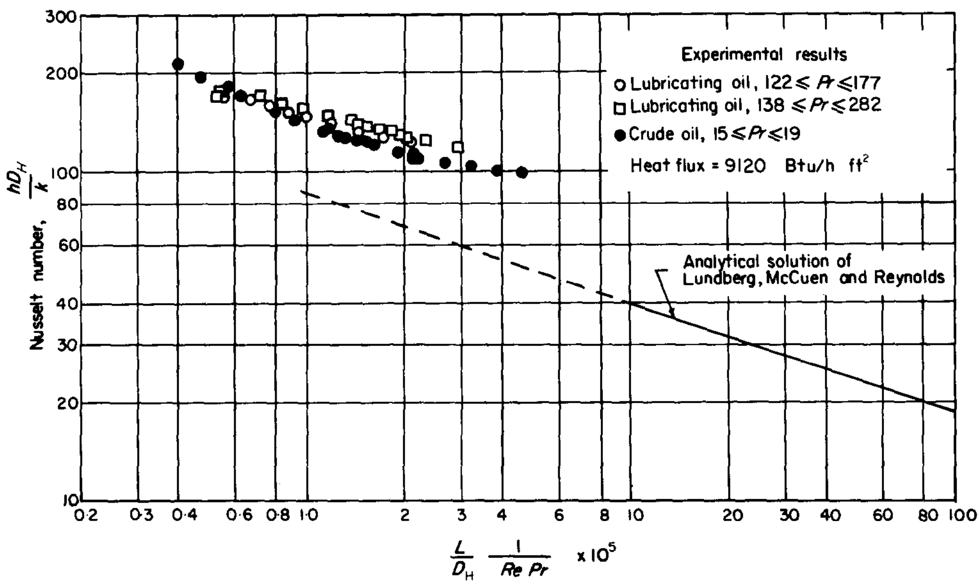

FIG. 9. Comparison of the laminar flow results with analytical solution of Lundberg, McCuen and Reynolds.

these graphs is defined as hD_H/k to follow the convention of these authors.) The trends are in good agreement. But the experimental results are 60–90 per cent higher than the analytical solutions in laminar flow, and 25–50 per cent higher than the analytical solution in turbulent

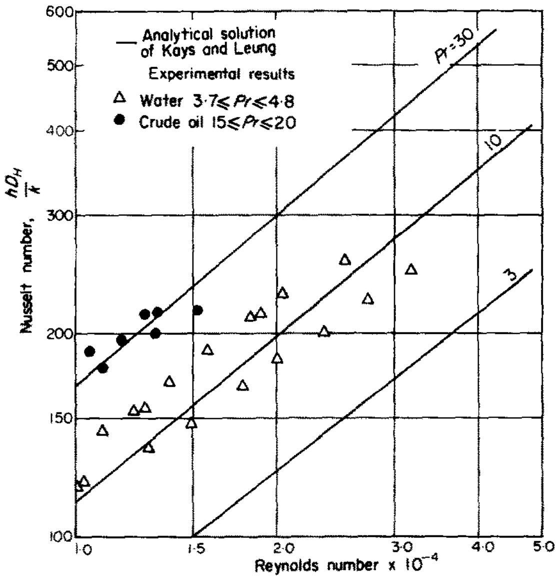

FIG. 10. Comparison of the turbulent flow results with analytical solutions of Kays and Leung.

flow. The discrepancy could result from the difference in the length of the heated section. It is consistent with heat transfer of laminar flow in tubes that a shorter heated section gives a higher heat-transfer coefficient. The comparison between different lengths of the heated section in turbulent flow (discussed later, Fig. 13) also seems to bear this out.

The general trend of the curve shown in Fig. 8 seems to agree with Mueller's results [9] of a heated wire inside a tube, with flow parallel to the wire. Note that the Nusselt number increases with an increasing rate as the Reynolds number is increased.

Effects of the diameter ratio of the annulus on the heat transfer of the annular flow were reported by Chen, Hawkins and Solberg [1] in the laminar flow regime, and by Monrad and Pelton [8] in the turbulent flow regime. Experimental results of the present work were reduced to the parameters suggested by these investigators and comparisons were made in Figs. 11 and 12. The consistency among these results indicates that the effect of diameter ratio on heat transfer depends on the Reynolds number of the flow; Chen *et al.* showed that in laminar flow the Nusselt number is proportional to $(D_2/D_1)^{0{\cdot}80}$, while Monrad and Pelton showed that in turbulent flow the Nusselt number is proportional to $(D_2/D_1)^{0{\cdot}53}$.

Farman and Beckmann [4], in a study of the heat transfer of turbulent, annular flow, varied the length of the heated section and suggested that the Nusselt number is proportional to a dimensionless length of the heated section raised to a power Ψ which depends on the Reynolds number of the flow. Turbulent flow results of the present experiment are compared with those of Farman and Beckmann in Fig. 13. The trend seems to be consistent. The increased value of heat-transfer coefficient as the ratio of L'/D_1 decreases also bears out the discrepancy indicated on Fig. 10. Moreover, Farman and Beckmann had observed the critical distance for turbulent flow to be less than 12·5 hydraulic diameters, which seems comparable with the present observation.

CONCLUSIONS

From the experimental results and the discussion, the following conclusions can be drawn:

In an annular flow system where a short section of the core of the annulus is heated and the length of this heated section is comparable either to the core diameter or to the hydraulic diameter of the annulus, heat transfer from the heated core is affected by location if the section is within a critical distance from the flow entry. This critical distance is approximately one-sixth of the hydrodynamic entrance length in the case of laminar flow, and ten hydraulic diameters in the case of turbulent flow.

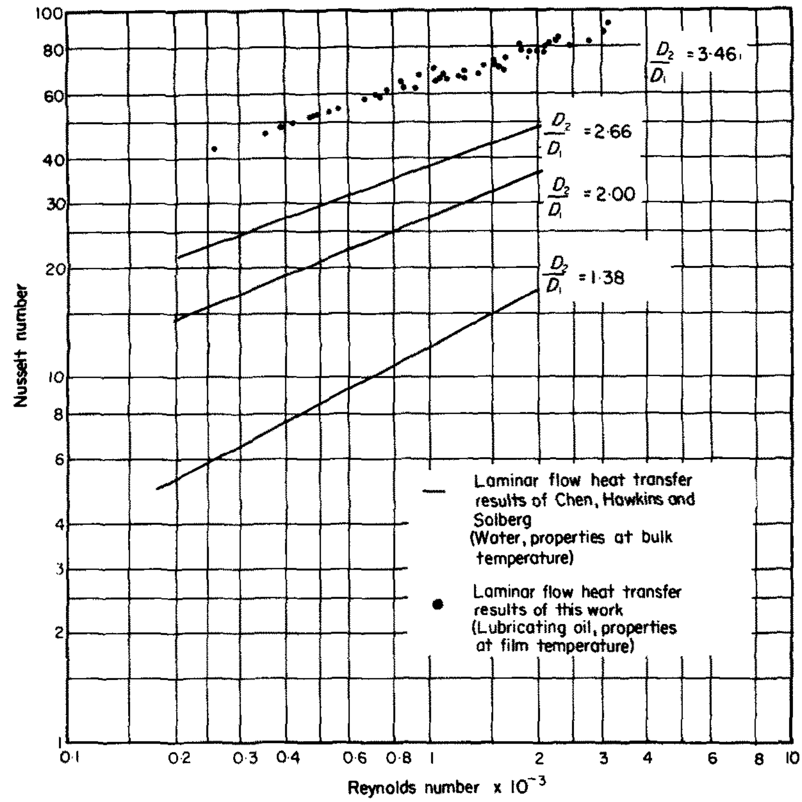

FIG. 11. Comparison of the laminar flow results with that of Chen, Hawkins and Solberg.

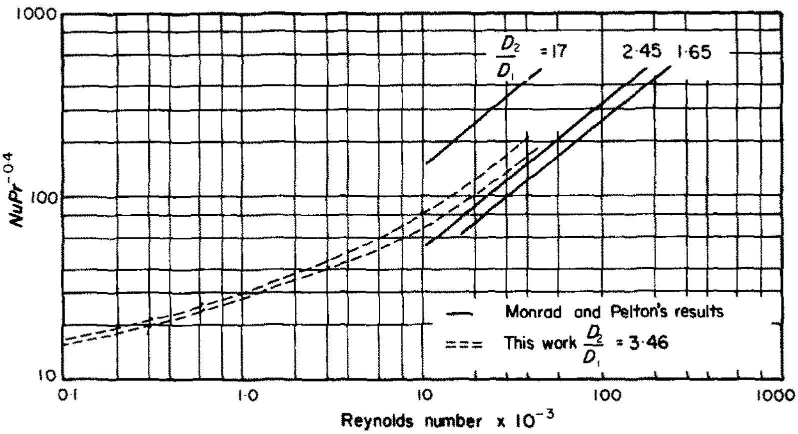

FIG. 12. Comparison of experimental results with Monrad and Pelton's results.

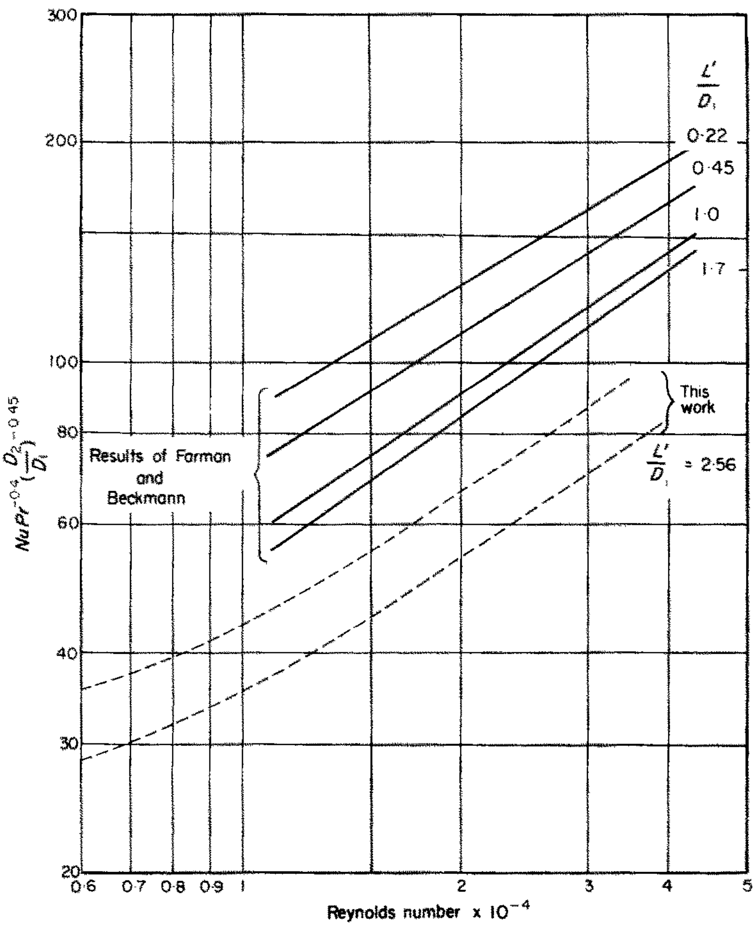

FIG. 13. Comparison of experimental results with Farman and Beckmann's results.

Beyond the critical distance, heat transfer resembles that for flow normal to a cylinder. The heat-transfer correlation takes the form

$$Nu = \alpha Re^{\beta}\left(\frac{D_2}{D_1}\right)^{\gamma}\left(\frac{L'}{D_1}\right)^{\varphi} Pr^{0.30}\left(\frac{\mu_w}{\mu}\right)^{0.14}$$

with α, β, γ and φ as functions of Reynolds number.

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Résumé—On a étudié expérimentalement le transport de chaleur à partir d'une courte portion chauffée de la partie centrale d'un système à écoulement annulaire vertical. Parmi les paramètres variables, se trouvaient l'enroit et le flux de chaleur de la portion chauffée, le type du fluide, le nombre de Prandtl du fluide et le nombre de Reynolds de l'écoulement.

Les résultats des essais ont montré que les coefficients de transport de chaleur étaient plus élevés lorsque la portion chauffée est située à une faible distance de l'entrée de l'écoulement. Cette distance est approximativement un sixième de la longueur d'entrée hydrodynamique pour l'écoulement laminaire et dix diamètres hydrauliques pour l'écoulement turbulent. Au-delà de cette distance, le transport de chaleur ressemble étroitement à celui de l'écoulement normal à un cylindre, et les résultats des essais ont été corrélés en une courbe unique.

Zusammenfassung—Eine experimentelle Untersuchung wurde für den Wärmeübergang von einem kurzen beheizten Kernstück, eines senkrechten Ringraumsystems durchgeführt. Variiert wurden die Parameter.

Ort- und Wärmefluss des beheizten Stückes, Art der Flüssigkeit, Prandtl-Zahl der Flüssigkeit und Reynolds-Zahl der Strömung. Die Versuche ergaben hohe Wäarmeübergangskoeffizienten wenn sich der beheizte Bereich in kurzer Entfernung von der Einströmstelle befand. Diese Entfernung betrug etwa ein sechstel der hydrodynamischen Einlauflänge für laminare Strömung und zehn hydraulische Durchmesser für turbulente Strömung. In grösserer Entfernung gleicht der Wärmeübergang stark dem bei Queranströkung eines Zylinders und die Versuchsergebnisse wurden in einer einzigen Kurve korreliert.

Аннотация—Проведено экспериментальное исследование с целью изучения переноса тепла от короткого нагретого участка на внутренней поверхности вертикальной кольцевой трубы. К изменяемым параметрам относились местоположение нагретого участка, плотность теплового потока, род жидкости, числа Прандтля и Рейнольдса.

Результаты опытов показали, что коэффициенты теплообмена имеют более высокие значения, когда нагретый участок находится вблизи входного участка. Это расстояние равно приблизительно $\frac{1}{6}$ длины гидродинамического входного участка при ламинарном течении и десяти гидравлическим диаметрам при турбулентном течении. За пределами этого расстояния поток тепла оказывается таким же, как в случае поперечно обтекаемого цилиндра, и результаты опытов на графике изображаются одной кривой.